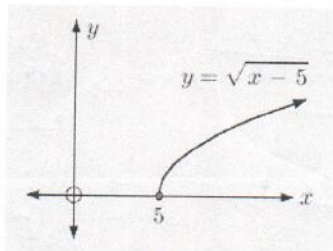




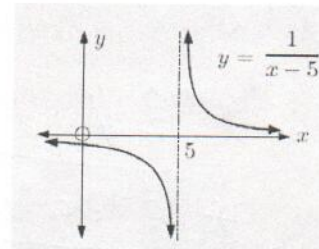
NOMBRE: _____ PROF: _____

1. Escriba el dominio y el rango de cada función.



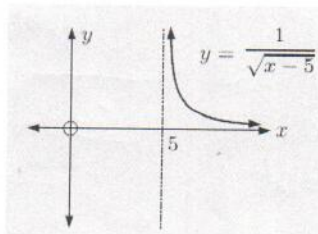
Dominio $(5, +\infty)$

Rango $(0, +\infty)$



Dominio $(-\infty, 5) \cup (5, +\infty)$

Rango $(-\infty, 0) \cup (0, +\infty)$



Dominio $(5, +\infty)$

Rango $(0, +\infty)$

2. Determine el dominio para cada una de las siguientes funciones:

a $f: x \mapsto 2x - 1$ \mathbb{R}

b $f(x) = 3$ \mathbb{R}

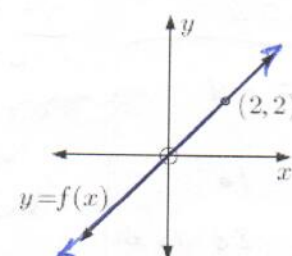
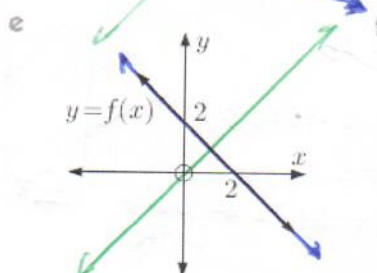
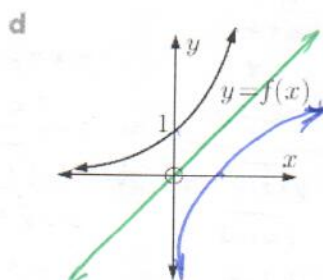
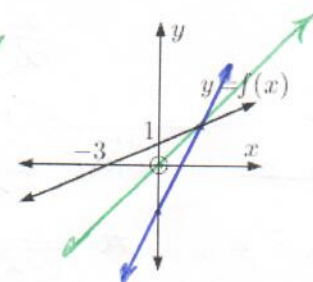
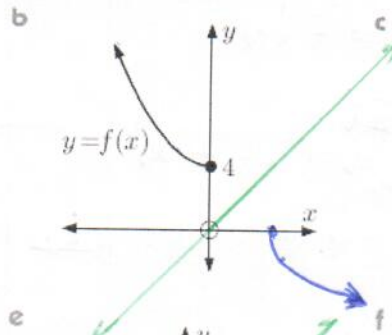
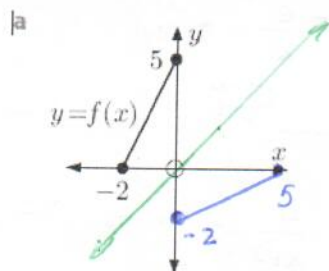
c $f: x \mapsto \sqrt{x}$ $x \geq 0$

d $f(x) = \frac{1}{x+1}$ $\mathbb{R} - \{-1\}$

e $f(x) = -\frac{1}{\sqrt{x}}$ $x > 0$

f $f: x \mapsto \frac{1}{3-x}$ $\mathbb{R} - \{3\}$

3. Grafique la inversa de cada una de las siguientes funciones.



4. De los siguientes ejercicios, NO RESUELVA el ejercicio 13.

✓ 10 Show that $f: x \mapsto \frac{3x-8}{x-3}$, $x \neq 3$ is a self-inverse function by:

a reference to its graph b using algebra.

✓ 11 Consider the function $f(x) = \frac{1}{2}x - 1$.

a Find $f^{-1}(x)$.

b Find: i $(f \circ f^{-1})(x)$ ii $(f^{-1} \circ f)(x)$.

✓ 12 Consider the functions $f: x \mapsto 2x+5$ and $g: x \mapsto \frac{8-x}{2}$.

a Find $g^{-1}(-1)$.

b Show that $f^{-1}(-3) - g^{-1}(6) = 0$.

c Find x such that $(f \circ g^{-1})(x) = 9$.

13 Consider the functions $f: x \mapsto 5^x$ and $g: x \mapsto \sqrt{x}$.

a Find: i $f(2)$ ii $g^{-1}(4)$. b Solve the equation $(g^{-1} \circ f)(x) = 25$.

✓ 14 Given $f: x \mapsto 2x$ and $g: x \mapsto 4x-3$, show that $(f^{-1} \circ g^{-1})(x) = (g \circ f)^{-1}(x)$.

✓ 15 Which of these functions is a self-inverse function?

a $f(x) = 2x$

b $f(x) = x$

c $f(x) = -x$

d $f(x) = \frac{2}{x}$

e $f(x) = -\frac{6}{x}$

5. Sean las funciones $f(x) = \frac{2x-4}{2}$, $g(x) = 6-x^2$ y $h(x) = x^2 - 3x + 4$, encuentre los valores de x para los que se cumple que:

a. $f(x) = 5$

b. $g(x) + h(x) = 6$

c. $g(x) = 0$

d. $h(x) = 6$

a. $\frac{2x-4}{2} = 5$
 $2x-4 = 10$
 $x = 7 //$

b. $(6-x^2) + (x^2-3x+4) = 6$
 $-3x+10 = 6$
 $-3x = -4$
 $x = \frac{4}{3} //$

c. $6-x^2 = 0$
 $6 = x^2$
 $x = \pm \sqrt{6} //$

d. $x^2-3x+4 = 6$
 $x^2-3x-2 = 0$
 $x = 3.56 \vee x = -0.56 //$

6. Determine $f^{-1}(0)$, $f^{-1}(-4)$, $f^{-1}(4)$ para cada una de las siguientes funciones:

a. $f(x) = \frac{x}{\sqrt{8-x}}$ b. $f(x) = \frac{4}{x+2} - 3$ c. $f(x) = \sqrt{x+10} - 4$

7. Para cada una de la siguientes funciones $f(x)$

a. $f(x) = \frac{2}{\sqrt{3x-5}}$ b. $f(x) = -x^2 - 5x, x \geq -2.5$ c. $f(x) = 3x+3$ d. $f(x) = -\sqrt{8-2x}$

a. $x > \frac{5}{3}, y > 0$

b. $x \geq -2.5, y \leq 6.25$

c. $x \in \mathbb{R}, y \in \mathbb{R}$

d. $x \leq 4, y \leq 0$

(i) Determine su dominio.

(ii) Dibuje su gráfica para los siguientes valores de $-5 \leq x \leq 5$, $-5 \leq y \leq 5$. Graficar asíntotas en caso de que existan.

(iii) Grafique la función inversa a partir del gráfico de $f(x)$.

(iv) Escriba dominio y rango de f^{-1} .

a. $x > 0, y > \frac{1}{2}$

b. $y \geq -2.5, x \leq 6.25$

c. $y \in \mathbb{R}, x \in \mathbb{R}$

d. $y \leq 4, x \leq 0$

8. Resuelva las siguientes ecuaciones:

a. $(2x+1) \cdot \frac{(2-5x)}{3} = 2$

b. $\frac{5x-2}{3} - 3x = \frac{x+1}{6}$

c. $\frac{5x-2}{3} - 3x = \frac{x+1}{6}$

d. $6-2x = 2 + \frac{(2-5x)}{3}$

e. $\frac{x-12}{4} - \frac{x+1}{2} = 0$

f. $\frac{x-2}{10} - x = \frac{2x-1}{5} + 1$

a. $2x-10x^2+2-5x = 6$
 $-10x^2-x-4 = 0$
 $x = 1.3 //$

b. $2(5x-2) - 6x = x+1$
 $10x-4-6x = x+1$
 $4x-4 = x+1$
 $3x = 5$
 $x = \frac{5}{3} //$

c. $2(5x-2) - 18x = x+1$
 $10x-4-18x = x+1$
 $-9x = 5$
 $x = -\frac{5}{9} //$

d. $6-2x = \frac{6+2-5x}{3}$
 $18-6x = 8-5x$
 $-x = -10$
 $x = 10 //$

e. $(x-12) - 2(x+1) = 0$
 $x-12-2x-2 = 0$
 $-x-14 = 0$
 $x = -14 //$

f. $(x-2) - 10x = \frac{2(2x-1)}{5} + 1$
 $-9x-2 = \frac{4x-2}{5} + 1$
 $-45x-10 = 4x-2+5$
 $-49x = 13$
 $x = -\frac{13}{49} //$

14. $f(x) = 2x + 1$ $g(x) = 4x - 3$
 mostre que $f^{-1} \circ g^{-1}(x) = (g \circ f)^{-1}(x)$

$$\rightarrow f(f^{-1}(x)) = x$$

$$2f^{-1}(x) = x$$

$$f^{-1}(x) = \frac{x}{2} //$$

$$\rightarrow g(g^{-1}(x)) = x$$

$$4g^{-1}(x) - 3 = x$$

$$g^{-1}(x) = \frac{x+3}{4} //$$

$$\star \boxed{f^{-1} \circ g^{-1}(x) = (g \circ f)^{-1}(x)}$$

$$\frac{g^{-1}(x)}{2} = \frac{\frac{x+3}{4}}{2}$$

$$\frac{x+3}{8} = \frac{\frac{x+3}{4}}{2} //$$

Si se cumple.

$$\rightarrow g \circ f(x) = 4f(x) - 3$$

$$= 4(2x) - 3$$

$$= 8x - 3 //$$

$$\rightarrow \boxed{g \circ f^{-1}(x) = y} //$$

$$\rightarrow g \circ f((g \circ f)^{-1}(x)) = x$$

$$g \circ f(y) = x$$

$$8y - 3 = x$$

$$y = \frac{x+3}{8}$$

$$(g \circ f)^{-1}(x) = \frac{x+3}{8} //$$

15.

a. $f(x) = 2x$

$$f \circ f^{-1}(x) = x$$

$$2f^{-1}(x) = x$$

$$f^{-1}(x) = \frac{x}{2}$$

NO

b. $f(x) = x$

$$f \circ f^{-1}(x) = x$$

$$f^{-1}(x) = x$$

Si

c. $f(x) = -x$

$$f \circ f^{-1}(x) = x$$

$$-f^{-1}(x) = x$$

$$f^{-1}(x) = -x$$

Si

d. $f(x) = \frac{2}{x}$

$$f \circ f^{-1}(x) = x$$

$$\frac{2}{f^{-1}(x)} = x$$

$$\frac{2}{x} = f^{-1}(x)$$

e. $f(x) = -\frac{6}{x}$

$$f \circ f^{-1}(x) = x$$

$$-\frac{6}{f^{-1}(x)} = x$$

$$-\frac{6}{x} = f^{-1}(x)$$

Si

tema 6: $f^{-1}(a) = x \Rightarrow a = f(x)$

a. $\frac{x}{\sqrt{8-x}} = 0$ $\star \frac{x}{\sqrt{8-x}} = 4$

$$x = 0 //$$

$$\star \frac{x}{\sqrt{8-x}} = -4$$

$$\frac{x^2}{8-x} = 16$$

$$x^2 = 128 - 16x$$

$$x^2 + 16x - 128 = 0$$

$$\begin{cases} x_1 = 5.86 \\ x_2 = -21.86 \end{cases} \rightarrow \text{NO cumple}$$

$$\frac{x^2}{8-x} = 16$$

$$x^2 + 16x - 128 = 0$$

$$\begin{cases} x_1 = 5.86 \\ x_2 = -21.86 \end{cases} \leftarrow \text{NO cumple}$$

b. $0 = \frac{4}{x+2} - 3$

$$3 = \frac{4}{x+2}$$

$$3x + 6 = 4$$

$$x = \frac{-2}{3} //$$

c. $0 = \sqrt{x+10} - 4$

$$4 = \sqrt{x+10}$$

$$16 = x + 10$$

$$6 = x //$$

$$\rightarrow -4 = \frac{4}{x+2} - 3 \Rightarrow 4 = \frac{4}{x+2}$$

$$-1 = \frac{4}{x+2}$$

$$-x - 2 = 4$$

$$-6 = x$$

$$\rightarrow x + 14 = 4$$

$$x = -10 //$$

$$0 = \sqrt{x+10} - 4$$

$$4 = \sqrt{x+10}$$

$$16 = x + 10$$

$$6 = x //$$

10

$$f(x) = \frac{3x-8}{x-3}$$

$$b. f(f^{-1}(x)) = x$$

$$\frac{3f^{-1}(x)-8}{f^{-1}(x)-3} = x$$

$$3f^{-1}(x)-8 = x f^{-1}(x)-3x$$

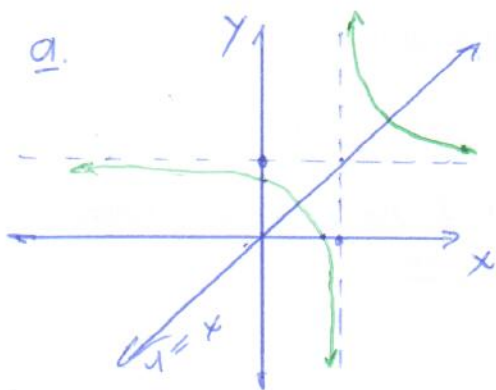
$$3f^{-1}(x)-x f^{-1}(x) = 8-3x$$

$$f^{-1}(x)(3-x) = 8-3x$$

$$f^{-1}(x) = \frac{8-3x}{3-x}$$

$$f^{-1}(x) = \frac{3x-8}{x-3}$$

$$f^{-1}(x) = f(x) \leftarrow \text{self-inverse}$$



$$11. f(x) = \frac{1}{2}x - 1$$

$$a. f^{-1}(x) \rightarrow f(f^{-1}(x)) = x$$

$$\frac{1}{2}f^{-1}(x) - 1 = x$$

$$f^{-1}(x) = 2(x+1)$$

$$b. f \circ f^{-1}(x) = x$$

$$c. f^{-1} \circ f(x) = x$$

$$12. f(x) = 2x+5 \quad \text{and} \quad g(x) = \frac{8-x}{2}$$

$$a. g^{-1}(x) \rightarrow g(g^{-1}(x)) = x$$

$$\frac{8-g^{-1}(x)}{2} = x$$

$$8-g^{-1}(x) = 2x$$

$$g^{-1}(x) = 8-2x$$

$$g^{-1}(-1) = 8+2 = 10$$

$$b. f^{-1}(-3) - g^{-1}(6) = 0$$

$$\rightarrow f^{-1}(-3) = x = 1$$

$$-3 = f(x)$$

$$-3 = 2x+5$$

$$2 = 2x$$

$$x = 1$$

$$\rightarrow g^{-1}(6) = \frac{8-6}{2} = 1$$

$$\rightarrow f^{-1}(-3) - g^{-1}(6) = 1 - 1 = 0$$

$$0 = 0$$

$$c. \rightarrow f \circ g^{-1}(x) = f(g^{-1}(x))$$

$$= 2g^{-1}(x) + 5$$

$$= 2(8-2x) + 5$$

$$= 16 - 4x + 5$$

$$= 21 - 4x$$

$$\rightarrow f \circ g^{-1}(x) = 9$$

$$21 - 4x = 9$$

$$-4x = -12$$

$$x = 3$$